

# Electrical Transport in Nanoscale Systems

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## Description

This book provides an in-depth description of transport phenomena relevant to systems of nanoscale dimensions. The different viewpoints and theoretical approaches are critically discussed, and their basic physical assumptions and approximations clearly spelled out. Topics that are usually not covered in literature, like the information content in the measurement of currents, or the role of initial conditions in establishing a steady state, are considered. The modern use of density-functional theory, in both its static and dynamical formulations, is also discussed. The language of information theory is used throughout the book to quantify the amount of information one can gather from either the measurement of the current, or the various descriptions of electrical conduction. The topics are introduced by simple physical arguments with particular attention to the non-equilibrium statistical nature of electrical conduction, and followed by a detailed formal derivation. This text will be of value to graduate students in physics, chemistry, and electrical engineering, as well as to researchers.

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## About the Author

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