MemComputing
Fundamentals and Applications

Clarifications and corrections

Chapter 3

- In Sec. 3.3 (Advantages of non-quantum systems (for computing)), in the margin note on "Degrees of freedom", it is written “they are the [maximum] number of ‘free’ variables that characterize the dynamics of the system”. The word “maximum” should be replaced by “minimal”.
- In Sec. 3.7 (Critical Points) it is written that the stability of an equilibrium point with center manifolds is only governed by the stable directions. This is true for digital MemComputing machines (see, e.g., F.L. Traversa, and M. Di Ventra, Chaos: An Interdisciplinary Journal of Nonlinear Science, 27, 023107 (2017)). However, it is not true for general dynamical systems. This is because spectral stability (determined by the eigenvalues of the Jacobian, Eq. (3.9)) does not necessarily imply general stability (see, e.g., J.E. Marsden, and T.S. Ratiu, Introduction to Mechanics and Symmetry (Springer, 1999)).

Chapter 4

- In footnote 11 of Chapter 4 it is written that for the liquid-gas transition “at the critical point, no change of symmetry occurs”. More precisely, it should be “*across* the critical point, no change of symmetry occurs”. This is because the liquid and gas phases have the same symmetry (translation invariance), but exactly *at* the critical point phase separation occurs, so that translation invariance is broken.

Chapter 7

- There is a typo in eqn (7.3): q_j should be in place of p_j. That equation should read:

$$q = \sum_{j=0}^{N-1} q_j 2^j, \tag{7.3}$$

- Note that the Example 7.3 of SO-AND gate in Chapter 7, as represented in the circuit diagram of Fig. 7.12 , may have additional stable critical points (for a certain class of initial conditions) that do not satisfy Boolean logic. These cases are easily removed by adding voltage-controlled differential-current generators as discussed in Example 7.4. The choice of such a 'reduced' SOLG was made in Ref. [Bearden, *et al.* Physical Review Applied, 9, 034029 (2018)] so that the dimensionality of the phase space would render the numerical analysis easier to carry out, in particular the search of critical points (a non-trivial task in multi-dimensional phase spaces).

Chapter 11

- In Section 11.3, it should be added that the price one pays to transition from the *finite*, and typically non-linear phase space of non-quantum dynamical systems to the (linear) Hilbert space of their algebraic description is that this Hilbert space is *infinite*. In other words, in order to represent a non-linear system (whether finite or infinite) linearly, one needs to render the ensuing vector space infinite-dimensional. This is in addition to the pseudo-Hermitian property of the evolution operator discussed in Section 11.3.

- In footnote 20 of Chapter 11, it is written that 'The dimension (dim σ) of the space of parameters σ (the moduli space) is equal to the index of the initial critical point of the instanton...'. This is true only if the final critical point of the instanton has zero index. In general, $\dim \sigma = \mathrm{ind}(\mathbf{x}^{\mathbf{i}}_{\mathbf{cr}}) - \mathrm{ind}(\mathbf{x}^{\mathbf{f}}_{\mathbf{cr}})$ ('ind' stands for 'index').

- In footnote 21 of Chapter 11, I make the point that instantons in DMMs 'connect a *neighborhood* of one critical point to the *neighborhood* of another'. This statement refers to the 'practical' realization of DMMs, whether in hardware or software, when (physical or numerical) noise cannot be avoided. Of course, from a strictly mathematical point of view, a critical point can always be reached *exactly* by an instanton, even if that critical point has unstable directions.

- In footnote 29 of Chapter 11, "*global minima*" should be "*local minima*". Instantons are *local* minima of the Euclidean action S_{DMM}.